On the Optimal Delay Amplification Factor of Multi-Hop Relay Channels

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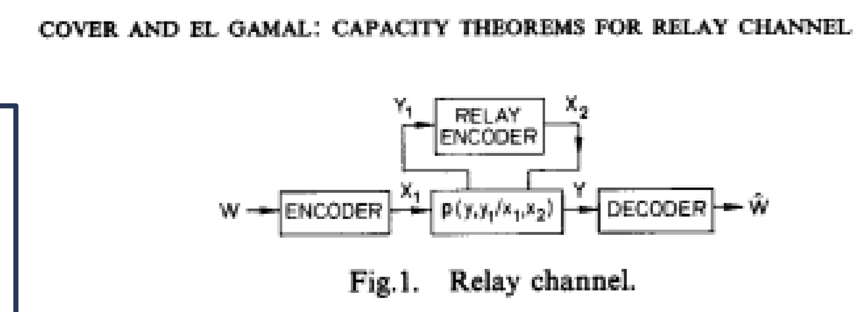
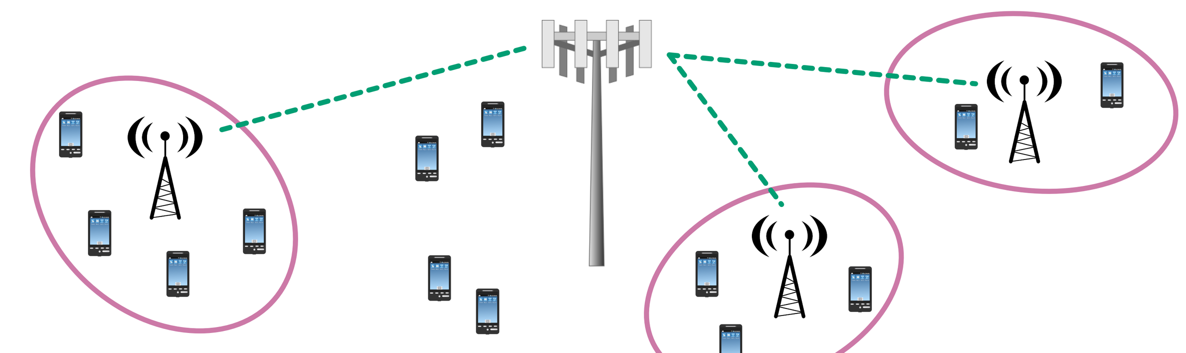
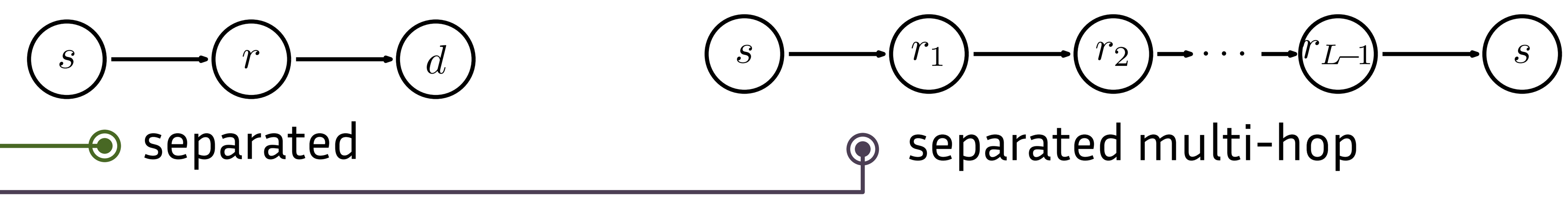
Motivation:

The **relay channel** is a classic information theory problem which has experienced a renewed surge of interest due to its applicability to a vast variety of **modern communication systems** (IoT, integrated access & backhaul, etc.)

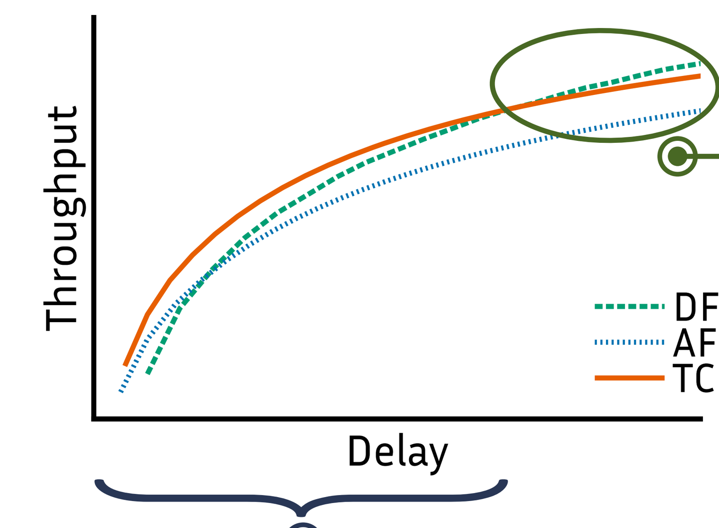




Many practical scenarios can be modeled as a simple **separated relay channel**. When three or more channels are concatenated, we speak of a **multi-hop** relay channel. Simple min-cut analysis shows that for multi-hop relays $C = \min\left(\{C_l\}_{l=1}^L\right)$, which is achievable by the decode-&-forward (DF) policy.



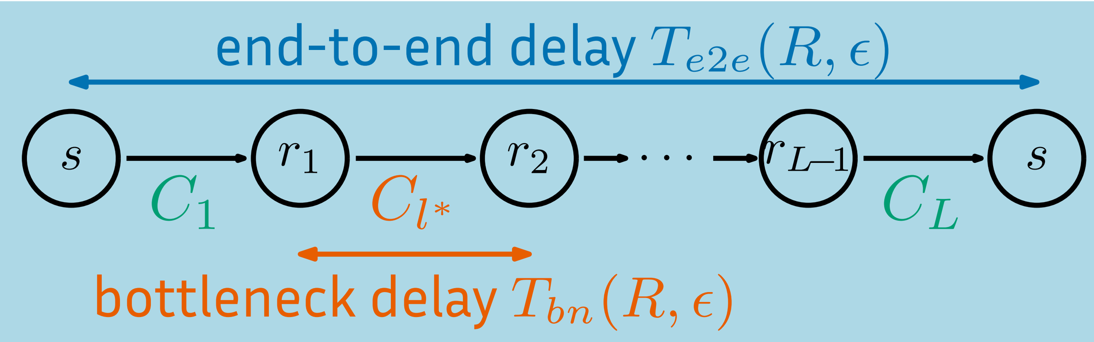
The recent push toward **low latency communications** (sub-1ms in IMT-2020 URLLC) motivates an investigation into the delay-throughput tradeoff for fixed error probabilities—finite blocklength theory.



Our work in [1] devised a new relaying scheme called **transcoding** (TC) that **substantially outperforms both DF and amplify-&-forward (AF) in the finite blocklength regime.** However, while transcoding outperforms DF for short block lengths, DF still has the upper hand in the **asymptotic regime.** To address this problem, this work presents two new schemes based on transcoding which outperform DF in a delay-throughput sense in the regime of asymptotically large, yet still finite block lengths.

Delay Amplification Factor:

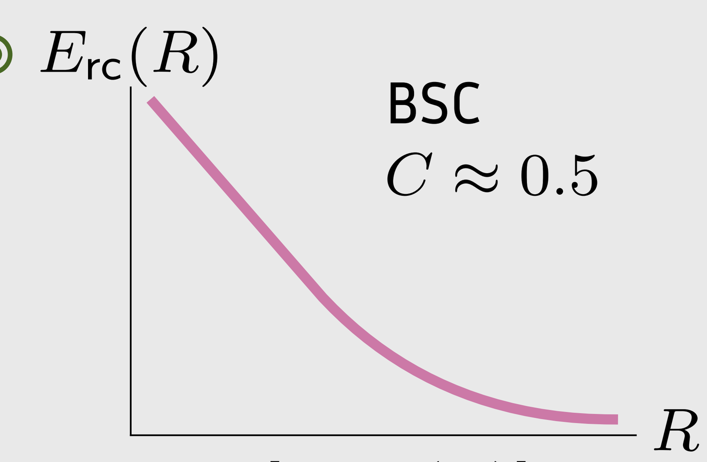
The main tool in showing this performance increase is our definition of the **Delay Amplification Factor (DAF)**, a measure of the multiplicative increase in delay when the full network is compared to the bottleneck link. An intuitive explanation of the DAF is that it is essentially the **ratio of the end-to-end delay from source to destination to the delay over the bottleneck hop alone.**



$$\mathrm{DAF}_\Phi \triangleq \lim_{R\nearrow C}\lim_{\epsilon\to 0}\frac{T_{e2e}(R,\epsilon)}{T_{bn}(R,\epsilon)}$$

$$\mathrm{DAF}_\Phi \triangleq \lim_{R\nearrow C}\frac{E_{\mathrm{rc},l^*}(R)}{E_\Phi(R)}$$

Error exponent approximation



$$\epsilon \le \exp\left[-nE(R)\right]$$

$$E(R) \triangleq \lim_{n\to\infty}\frac{-\log(\epsilon)}{n}$$

$$T_{bn}(R,\epsilon) \gtrsim -\frac{1}{E_{\mathrm{rc},l^*}(R)}\log(\epsilon)$$

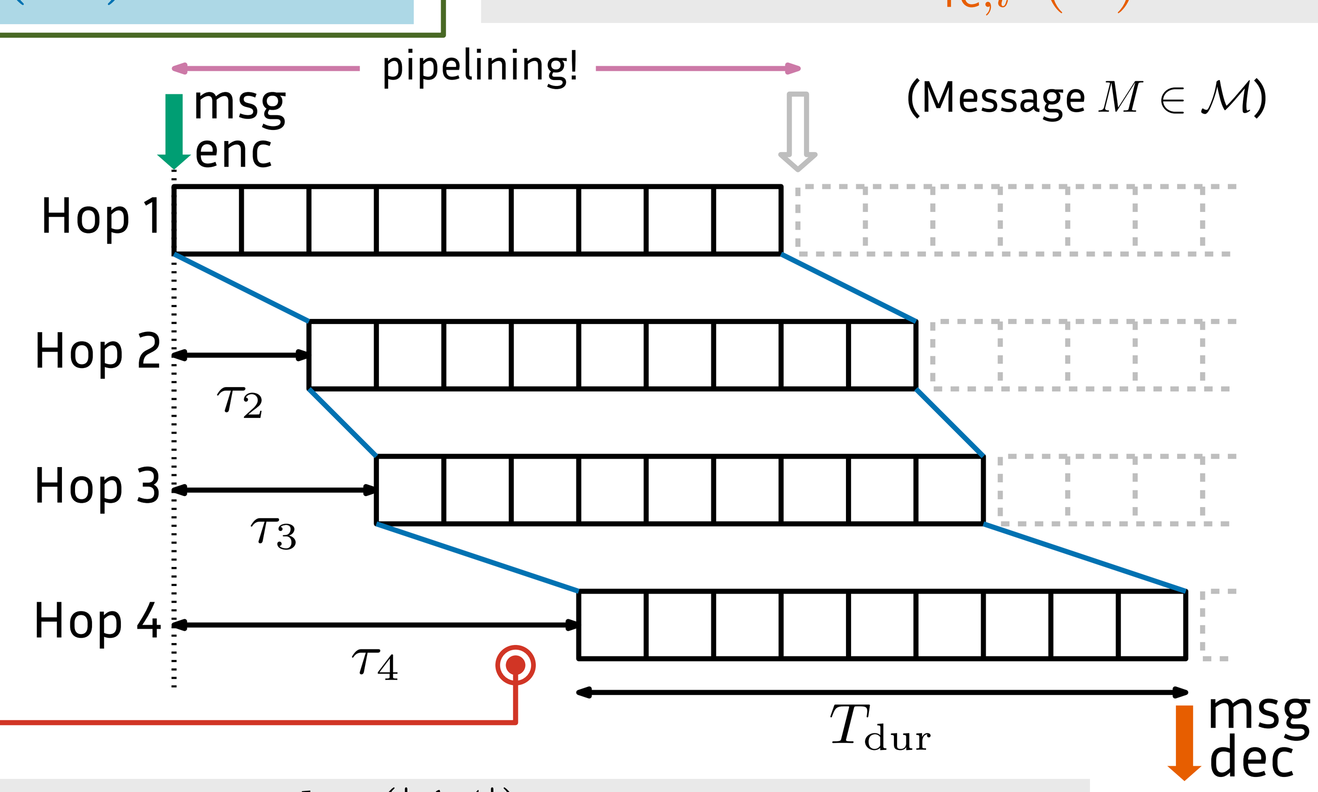
We analyze those delays using error exponents. We obtain the delay over the bottleneck hop from Gallager's **random coding error exponent.** The end-to-end delay depends on the specific relaying scheme used and is analyzed by deriving the **end-to-end error exponent** from a description of a relaying scheme. The DAF is then equivalently formulated as the limit of the ratio of the bottleneck and the end-to-end error exponents.

End-to-end error exponent

$E_\Phi(R)$: "Error exponent of relaying scheme Φ"

$$T_{e2e}(R,\epsilon) \gtrsim -\frac{1}{E_\Phi(R)}\log(\epsilon)$$

- Coding scheme
- Channel transition probabilities
- Operation at relays



Feasible (T,R,ϵ): (i) $T \ge \tau_L + T_{\mathrm{dur}}$, (ii) $R \le \frac{\log(|\mathcal{M}|)}{T_{\mathrm{dur}}}$, (iii) $\epsilon \ge P(M \neq \widehat{M})$

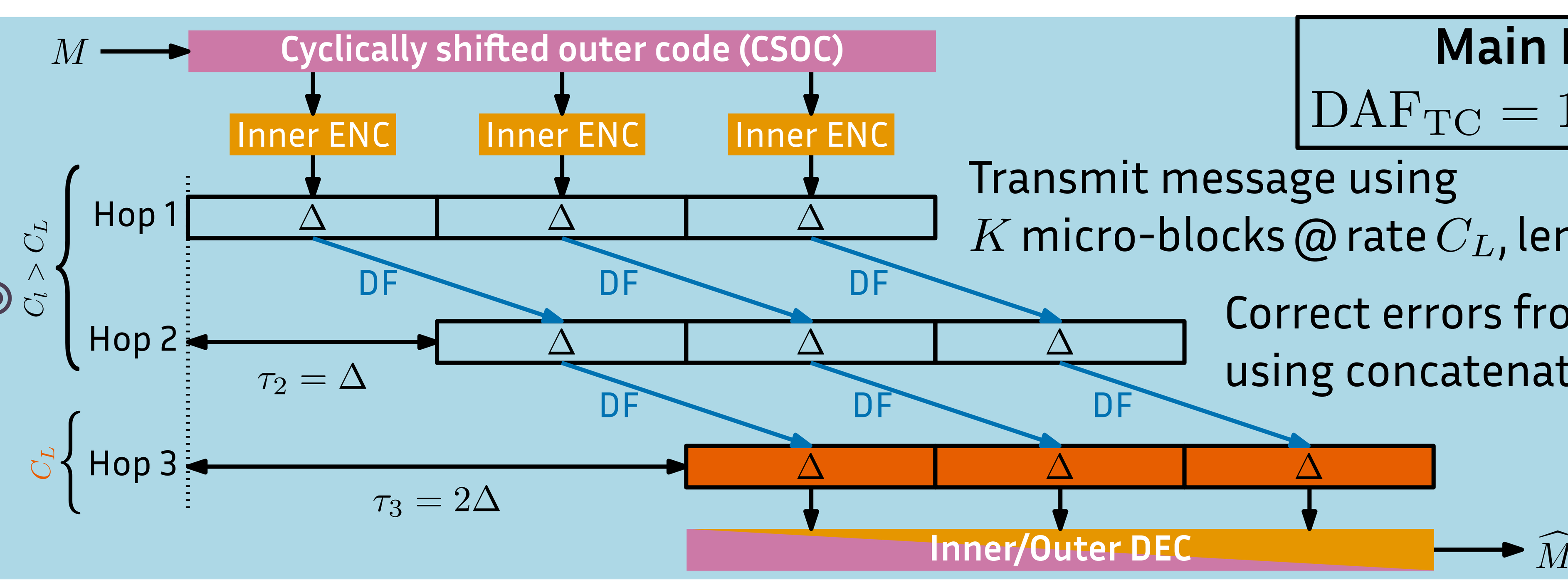
Then $E_\Phi(R) \triangleq \limsup_{T\to\infty}\ \sup_{\epsilon:(T,R,\epsilon)\text{ is feasible}}\frac{-\log(\epsilon)}{T}$

Main Results:

We show that the DAF for DF schemes is strictly larger than 1. For AF schemes, the DAF does not apply, since they are not capacity achieving.

In contrast to this, our **first main result** is an open-loop transcoding scheme that achieves DAF=1 when the bottleneck hop is the last hop ($l^* = L$). The proof hinges on a careful construction of a concatenated code and maximum-likelihood joint decoding at the destination

Main Result #1: $\mathrm{DAF}_{\mathrm{TC}} = 1$ **if** $l^* = L$

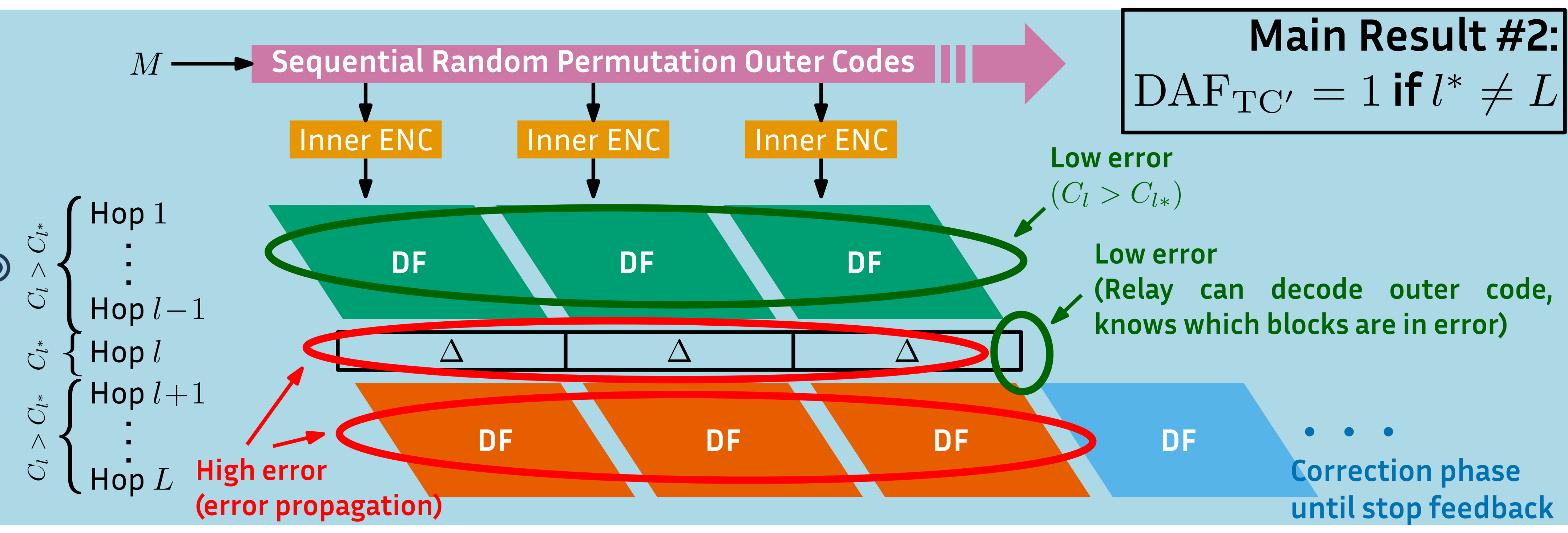


Transmit message using K micro-blocks @ rate C_L, length Δ

Correct errors from last hop using concatenated code

The **second main result** is a one-time stop feedback scheme that achieves DAF=1 regardless of the position of the bottleneck $l^* \neq L$. The proof builds on the idea that the relay knows which micro-blocks it decoded wrong after forwarding all of them.

Main Result #2: $\mathrm{DAF}_{\mathrm{TC}'} = 1$ **if** $l^* \neq L$



Using result #1, we can construct a sub-optimal ($1 \le \mathrm{DAF}_{\Phi'} \le \mathrm{DAF}_{\mathrm{DF}}$) open-loop scheme that outperforms DF for $l^* \neq L$. Our work in progress is investigating open-loop schemes that can do better.

Publications:

[1] C.-C. Wang, D. J. Love, and D. Ogbe, "Transcoding: A new strategy for relay channels," Allerton 2017, Oct. 2017.
[2] D. Ogbe, C.-C. Wang, and D. J. Love, "On the Optimal Delay Amplification Factor of Multi-Hop Relay Channels," ISIT 2019, July 2019.