On the Optimal Delay Amplification Factor of Multi-Hop Relay Channels

Dennis Ogbe, Chih-Chun Wang, David J. Love

School of Electrical and Computer Engineering
Purdue University
West Lafayette, Indiana, USA

International Symposium on Information Theory
Paris, France, July 12, 2019

This work was supported in parts by the National Science Foundation under Grant CCF-1422997, Grant ECCS-1407604, Grant CCF-1618475, and Grant CCF-1816013.

Overview

- **Part 1:** ***Motivation & Intuition***
  - Motivation
  - A new metric for multi-hop relay channels: *The Delay Amplification Factor*
  - Preview of main results
- **Part 2:** ***Main Results***
  - (Necessary) details of problem set-up
  - Main results & proof sketches
  - Work in progress & Conclusion

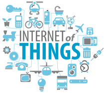
Overview

- **Part 1:** ***Motivation & Intuition***
  - Motivation
  - A new metric for multi-hop relay channels: *The Delay Amplification Factor*
  - Preview of main results
- **Part 2:** ***Main Results***
  - (Necessary) details of problem set-up
  - Main results & proof sketches
  - Work in progress & Conclusion

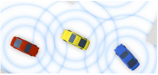
Motivation

Motivation

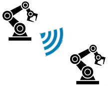
Low latency communications



IoT



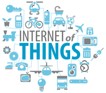
Autonomous driving



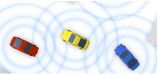
MTC

Motivation

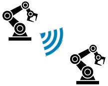
Low latency communications



IoT



Autonomous driving

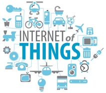


MTC

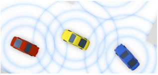
IMT-2020 URLLC: $\leq$ 1ms delay

Motivation

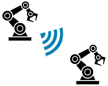
Low latency communications



IoT



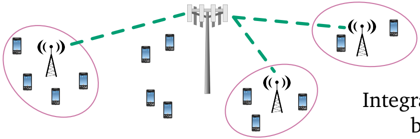
Autonomous driving



MTC

IMT-2020 URLLC: $\leq$ 1ms delay

Flexible/dense network architectures

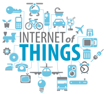


Small cells

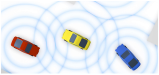
Integrated access & backhaul

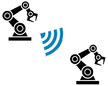
Motivation

Low latency communications



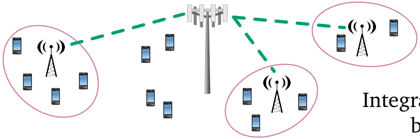
IoT


Autonomous driving


MTC

IMT-2020 URLLC: $\leq$ 1ms delay

Flexible/dense network architectures



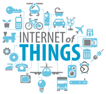
Small cells

Integrated access & backhaul

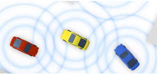
For IT: Lots of relay channels

Motivation

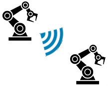
Low latency communications



IoT



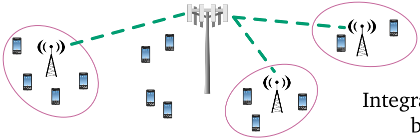
Autonomous driving



MTC

IMT-2020 URLLC: $\leq$ 1ms delay

Flexible/dense network architectures



Small cells

Integrated access & backhaul

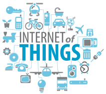
For IT: Lots of relay channels

- Recent technologies $\Rightarrow$ renewed interest in **delay-throughput tradeoff** of relay channels

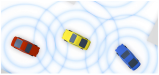
Dennis Ogbe, Chih-Chun Wang, David J. Love
On the Optimal Delay Amplification Factor of Multi-Hop Relay Channels
International Symposium on Information Theory
Paris, France, July 12, 2019

Motivation

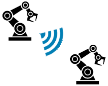
Low latency communications



IoT



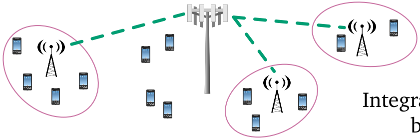
Autonomous driving



MTC

IMT-2020 URLLC: $\leq$ 1ms delay

Flexible/dense network architectures



Small cells

Integrated access & backhaul

For IT: Lots of relay channels

- Recent technologies $\Rightarrow$ renewed interest in **delay-throughput tradeoff** of relay channels
- Interesting from practical perspective: **Separated** relay channel over **multiple hops**

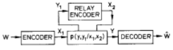
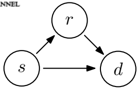
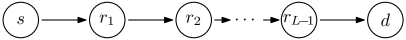

COVER AND EL GAMAL: CAPACITY THEOREMS FOR RELAY CHANNEL

Fig.1. Relay channel.



general

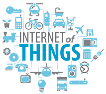
separated



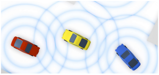
separated multi-hop

Motivation

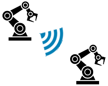
Low latency communications



IoT



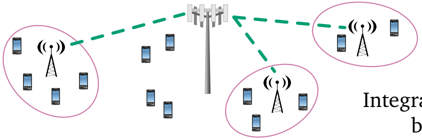
Autonomous driving



MTC

IMT-2020 URLLC: $\leq$ 1ms delay

Flexible/dense network architectures



Small cells

Integrated access & backhaul

For IT: Lots of relay channels

- Recent technologies $\Rightarrow$ renewed interest in **delay-throughput tradeoff** of relay channels
- Interesting from practical perspective: **Separated** relay channel over **multiple hops**

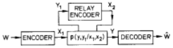
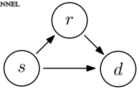

COVER AND EL GAMAL: CAPACITY THEOREMS FOR RELAY CHANNEL

Fig. 1. Relay channel.

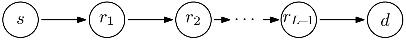


general

capacity $= \min(C_1, C_2)$

separated

capacity $= \min(C_1, C_2, \ldots, C_L)$



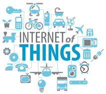
separated multi-hop

- Capacity analysis is clear, delay-throughput analysis is not

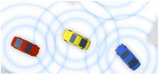
Dennis Ogbe, Chih-Chun Wang, David J. Love

Motivation

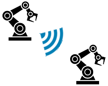
Low latency communications



IoT



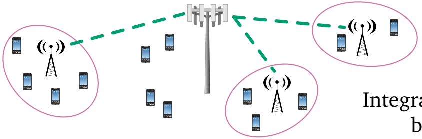
Autonomous driving



MTC

IMT-2020 URLLC: $\leq$ 1ms delay

Flexible/dense network architectures



Small cells

Integrated access & backhaul

For IT: Lots of relay channels

- Recent technologies $\Rightarrow$ renewed interest in **delay-throughput tradeoff** of relay channels
- Interesting from practical perspective: **Separated** relay channel over **multiple hops**

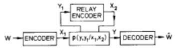
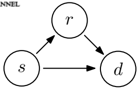

COVER AND EL GAMAL: CAPACITY THEOREMS FOR RELAY CHANNEL

Fig. 1. Relay channel.

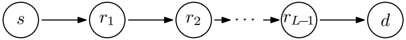


general

capacity $= \min(C_1, C_2)$

separated

capacity $= \min(C_1, C_2, \ldots, C_L)$

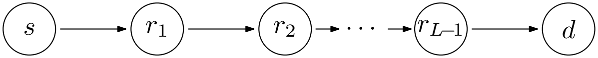


separated multi-hop

- Capacity analysis is clear, delay-throughput analysis is not
- **This work:** When $R \to C$, what relaying schemes minimize (relative) delay?

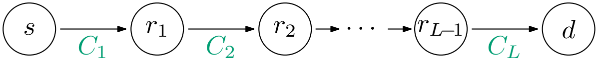
Dennis Ogbe, Chih-Chun Wang, David J. Love

A new metric: The Delay Amplification Factor



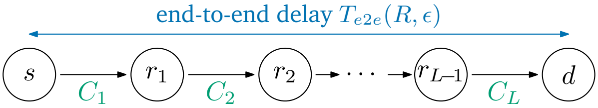
A new metric: The Delay Amplification Factor

$$C = \min\left(\{C_l\}_{l=1}^{L}\right)$$



A new metric: The Delay Amplification Factor

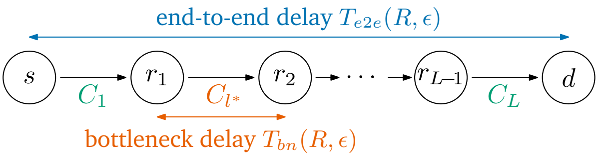
$$C = \min\left(\{C_l\}_{l=1}^{L}\right)$$



A new metric: The Delay Amplification Factor

$$C = \min\left(\{C_l\}_{l=1}^{L}\right) = C_{l^*}$$

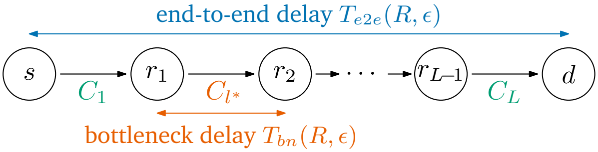
(assume unique C_{l^*})



A new metric: The Delay Amplification Factor

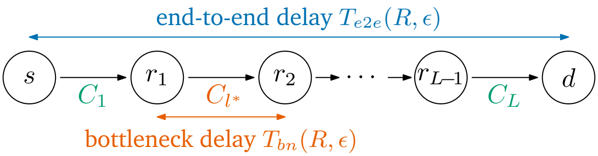
$$C = \min\left(\{C_l\}_{l=1}^{L}\right) = C_{l^*}$$

(assume unique C_{l^*})



$$\mathrm{DAF}_\Phi \triangleq \lim_{R \nearrow C} \lim_{\epsilon \to 0} \frac{T_{e2e}(R, \epsilon)}{T_{bn}(R, \epsilon)}$$

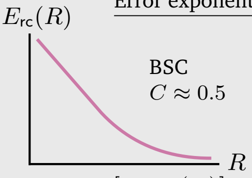
A new metric: The Delay Amplification Factor



$$C = \min\left(\{C_l\}_{l=1}^{L}\right) = C_{l^*}$$

(assume unique C_{l^*})

Error exponent approx. [Gallager, 1968][1]



$$\epsilon \leq \exp\left[-nE(R)\right]$$

$$E(R) \triangleq \lim_{n\to\infty} \frac{-\log(\epsilon)}{n}$$

$$T_{bn}(R, \epsilon) \gtrsim -\frac{1}{E_{\mathsf{rc},l^*}(R)} \log(\epsilon)$$

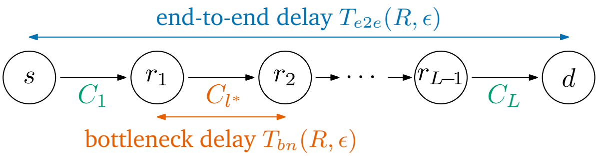
$$\mathrm{DAF}_\Phi \triangleq \lim_{R\nearrow C} \lim_{\epsilon\to 0} \frac{T_{e2e}(R, \epsilon)}{T_{bn}(R, \epsilon)}$$

[1] R. G. Gallager, *Information Theory and Reliable Communication*. New York, NY, USA: John Wiley & Sons, Inc., 1968.

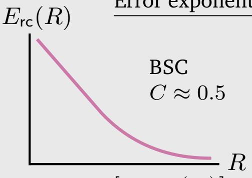
A new metric: The Delay Amplification Factor

$$C = \min\left(\{C_l\}_{l=1}^{L}\right) = C_{l^*}$$

(assume unique C_{l^*})



Error exponent approx. [Gallager, 1968][1]



$$\epsilon \leq \exp\left[-nE(R)\right]$$

$$E(R) \triangleq \lim_{n\to\infty} \frac{-\log(\epsilon)}{n}$$

$$T_{bn}(R,\epsilon) \gtrsim -\frac{1}{E_{\mathrm{rc},l^*}(R)} \log(\epsilon)$$

End-to-end error exponent

$$T_{e2e}(R,\epsilon) \gtrsim -\frac{1}{E_{\Phi}(R)} \log(\epsilon)$$

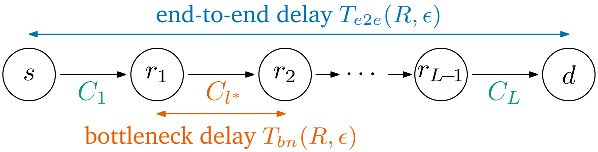
$$\mathrm{DAF}_{\Phi} \triangleq \lim_{R \nearrow C} \lim_{\epsilon\to 0} \frac{T_{e2e}(R,\epsilon)}{T_{bn}(R,\epsilon)}$$

[1] R. G. Gallager, *Information Theory and Reliable Communication*. New York, NY, USA: John Wiley & Sons, Inc., 1968.

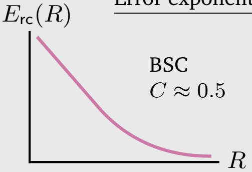
A new metric: The Delay Amplification Factor

$$C = \min\left(\{C_l\}_{l=1}^{L}\right) = C_{l^*}$$

(assume unique C_{l^*})



Error exponent approx. [Gallager, 1968]



$$\epsilon \le \exp\left[-nE(R)\right]$$

$$E(R) \triangleq \lim_{n\to\infty} \frac{-\log(\epsilon)}{n}$$

$$T_{bn}(R,\epsilon) \gtrsim -\frac{1}{E_{\mathsf{rc},l^*}(R)}\log(\epsilon)$$

End-to-end error exponent

$$T_{e2e}(R,\epsilon) \gtrsim -\frac{1}{E_\Phi(R)}\log(\epsilon)$$

$E_\Phi(R)$: "Error exponent of relaying scheme Φ"

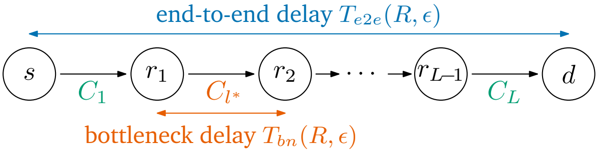
- Coding scheme
- Channel transition probabilities
- Operation at relays

} need more definitions

$$\mathrm{DAF}_\Phi \triangleq \lim_{R\nearrow C}\lim_{\epsilon\to 0}\frac{T_{e2e}(R,\epsilon)}{T_{bn}(R,\epsilon)}$$

[1] R. G. Gallager, *Information Theory and Reliable Communication*. New York, NY, USA: John Wiley & Sons, Inc., 1968.

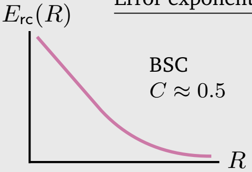
A new metric: The Delay Amplification Factor



$$C = \min\left(\{C_l\}_{l=1}^{L}\right) = C_{l^*}$$

(assume unique C_{l^*})

Error exponent approx. [Gallager, 1968]



$$\epsilon \leq \exp\left[-nE(R)\right]$$

$$E(R) \triangleq \lim_{n\to\infty} \frac{-\log(\epsilon)}{n}$$

$$T_{bn}(R,\epsilon) \gtrsim -\frac{1}{E_{\mathsf{rc},l^*}(R)}\log(\epsilon)$$

End-to-end error exponent

$$T_{e2e}(R,\epsilon) \gtrsim -\frac{1}{E_{\Phi}(R)}\log(\epsilon)$$

$E_{\Phi}(R)$: "Error exponent of relaying scheme Φ"

- Coding scheme
- Channel transition probabilities
- Operation at relays

} need more definitions

$$\mathrm{DAF}_{\Phi} \triangleq \lim_{R\nearrow C}\lim_{\epsilon\to 0} \frac{T_{e2e}(R,\epsilon)}{T_{bn}(R,\epsilon)}$$

or, equivalently

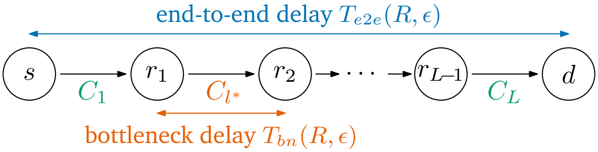
$$\mathrm{DAF}_{\Phi} \triangleq \lim_{R\nearrow C} \frac{E_{\mathsf{rc},l^*}(R)}{E_{\Phi}(R)}$$

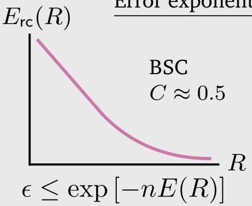
[1] R. G. Gallager, *Information Theory and Reliable Communication*. New York, NY, USA: John Wiley & Sons, Inc., 1968.

A new metric: The Delay Amplification Factor

$$C = \min\left(\{C_l\}_{l=1}^{L}\right) = C_{l^*}$$

(assume unique C_{l^*})





Error exponent approx. [Gallager, 1968]

$$E(R) \triangleq \lim_{n\to\infty} \frac{-\log(\epsilon)}{n}$$

$$T_{bn}(R,\epsilon) \gtrsim -\frac{1}{E_{\mathsf{rc},l^*}(R)} \log(\epsilon)$$

End-to-end error exponent

$$T_{e2e}(R,\epsilon) \gtrsim -\frac{1}{E_{\Phi}(R)} \log(\epsilon)$$

$E_{\Phi}(R)$: "Error exponent of relaying scheme Φ"

- Coding scheme
- Channel transition probabilities
- Operation at relays

} need more definitions

$$\mathrm{DAF}_{\Phi} \triangleq \lim_{R\nearrow C} \lim_{\epsilon\to 0} \frac{T_{e2e}(R,\epsilon)}{T_{bn}(R,\epsilon)}$$

or, equivalently

$$\mathrm{DAF}_{\Phi} \triangleq \lim_{R\nearrow C} \frac{E_{\mathsf{rc},l^*}(R)}{E_{\Phi}(R)}$$

*Small Lemma: $\mathrm{DAF}_{\Phi} \geq 1 \;\; \forall \Phi$

[1] R. G. Gallager, *Information Theory and Reliable Communication*. New York, NY, USA: John Wiley & Sons, Inc., 1968.

Preview of main results

Preview of main results

Definition

We define the *Delay Amplification factor of an L-hop relaying scheme* Φ as

$$\mathrm{DAF}_\Phi \triangleq \lim_{R \nearrow C} \frac{E_{\mathsf{rc},l^*}(R)}{E_\Phi(R)}.$$

Preview of main results

Definition

We define the *Delay Amplification factor of an L-hop relaying scheme* Φ as

$$\mathrm{DAF}_{\Phi} \triangleq \lim_{R \nearrow C} \frac{E_{\mathsf{rc},l^*}(R)}{E_{\Phi}(R)}.$$

Lemmas

1. Regardless of the relaying scheme, we always have $\mathrm{DAF} \geq 1$
2. For decode-&-forward schemes, we have $\mathrm{DAF}_{\mathrm{DF}} = \mathcal{O}(L)$

Preview of main results

Definition

We define the *Delay Amplification factor of an L-hop relaying scheme* Φ as

$$\mathrm{DAF}_{\Phi} \triangleq \lim_{R \nearrow C} \frac{E_{\mathsf{rc},l^*}(R)}{E_{\Phi}(R)}.$$

Lemmas

1. Regardless of the relaying scheme, we always have $\mathrm{DAF} \geq 1$
2. For decode-&-forward schemes, we have $\mathrm{DAF}_{\mathrm{DF}} = \mathcal{O}(L)$

Main results

1. A relaying scheme where $\mathrm{DAF}_{\Phi} = 1$ if $l^* = L$
2. A 1-bit stop feedback relaying scheme where $\mathrm{DAF}_{\Phi'} = 1$ regardless of the position of l^*

Overview

- Part 1: *Motivation & Intuition*
  - Motivation
  - A new metric for multi-hop relay channels: *The Delay Amplification Factor*
  - Preview of main results
- **Part 2: *Main Results***
  - (Necessary) details of problem set-up
  - Main results & proof sketches
  - Work in progress & Conclusion

Starting times, relay encoding & joint decoding

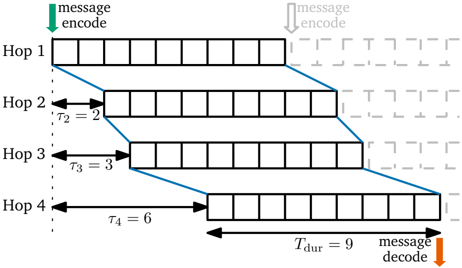
A relaying scheme is defined by

- Message $M \in \mathcal{M} \triangleq \{1, \ldots, |\mathcal{M}|\}$
- L deterministic starting times $\tau_1 = 0 \leq \tau_2 \leq \cdots \leq \tau_L$
- Transmission duration T_{dur}

Starting times, relay encoding & joint decoding

A relaying scheme is defined by

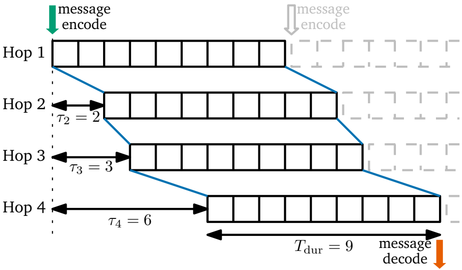
- Message $M \in \mathcal{M} \triangleq \{1, \ldots, |\mathcal{M}|\}$
- L deterministic starting times $\tau_1 = 0 \leq \tau_2 \leq \cdots \leq \tau_L$
- Transmission duration T_{dur}



Starting times, relay encoding & joint decoding

A relaying scheme is defined by

- Message $M \in \mathcal{M} \triangleq \{1, \dots, |\mathcal{M}|\}$
- L deterministic starting times $\tau_1 = 0 \le \tau_2 \le \cdots \le \tau_L$
- Transmission duration T_{dur}

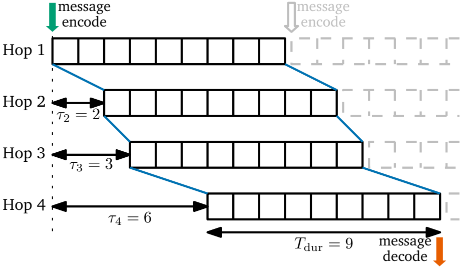


- Coding scheme

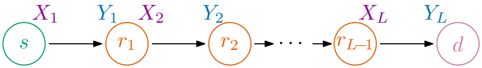
Starting times, relay encoding & joint decoding

A relaying scheme is defined by

- Message $M \in \mathcal{M} \triangleq \{1, \ldots, |\mathcal{M}|\}$
- L deterministic starting times $\tau_1 = 0 \le \tau_2 \le \cdots \le \tau_L$
- Transmission duration T_{dur}



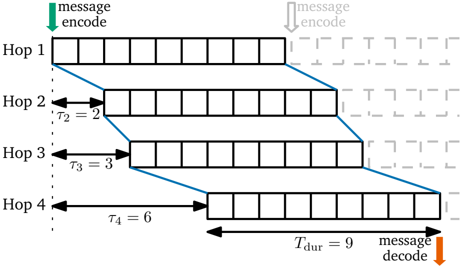
- Coding scheme



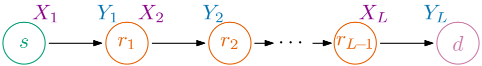
Starting times, relay encoding & joint decoding

A relaying scheme is defined by

- Message $M \in \mathcal{M} \triangleq \{1, \ldots, |\mathcal{M}|\}$
- L deterministic starting times $\tau_1 = 0 \leq \tau_2 \leq \cdots \leq \tau_L$
- Transmission duration T_{dur}



- Coding scheme



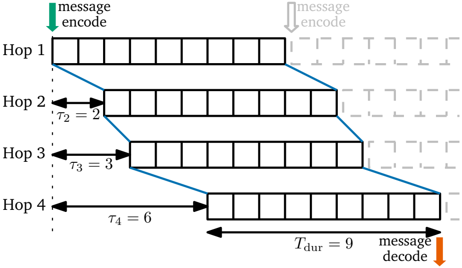
- Block encoding @ source

$$X_1(t) = f_t^{[1]}(M)$$

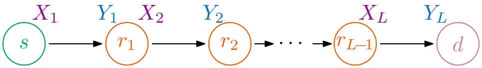
Starting times, relay encoding & joint decoding

A relaying scheme is defined by

- Message $M \in \mathcal{M} \triangleq \{1, \ldots, |\mathcal{M}|\}$
- L deterministic starting times $\tau_1 = 0 \leq \tau_2 \leq \cdots \leq \tau_L$
- Transmission duration T_{dur}



- Coding scheme



- Block encoding @ source

$$X_1(t) = f_t^{[1]}(M)$$

- Causal-sequential coding @ relays

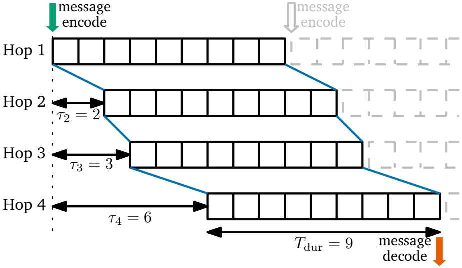
$$Y_l(t) = \mathrm{DMC}_l\big(X_l(t)\big)$$

$$X_l(t) = f_t^{[l]}\left([Y_{l-1}]_*^{t-1}\right)$$

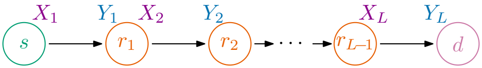
Starting times, relay encoding & joint decoding

A relaying scheme is defined by

- Message $M \in \mathcal{M} \triangleq \{1, \ldots, |\mathcal{M}|\}$
- L deterministic starting times $\tau_1 = 0 \le \tau_2 \le \cdots \le \tau_L$
- Transmission duration T_{dur}



- Coding scheme



- Block encoding @ source

$$X_1(t) = f_t^{[1]}(M)$$

- Causal-sequential coding @ relays

$$Y_l(t) = \text{DMC}_l\big(X_l(t)\big)$$

$$X_l(t) = f_t^{[l]}([Y_{l-1}]_*^{t-1})$$

- Block decoding @ destination

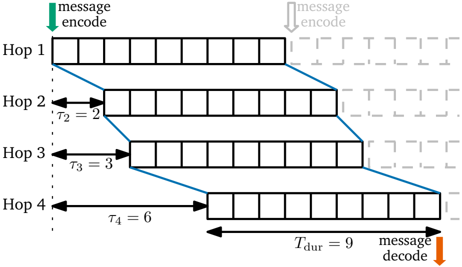
$$\widehat{M} = g([Y_L]_*^{\tau_L + T_{\text{dur}}})$$

Dennis Ogbe, Chih-Chun Wang, David J. Love

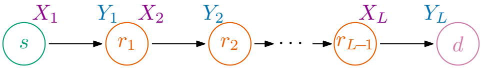
Starting times, relay encoding & joint decoding

A relaying scheme is defined by

- Message $M \in \mathcal{M} \triangleq \{1, \dots, |\mathcal{M}|\}$
- L deterministic starting times $\tau_1 = 0 \leq \tau_2 \leq \cdots \leq \tau_L$
- Transmission duration T_{dur}



- Coding scheme



- Block encoding @ source

$$X_1(t) = f_t^{[1]}(M)$$

- Causal-sequential coding @ relays

$$Y_l(t) = \text{DMC}_l\big(X_l(t)\big)$$

$$X_l(t) = f_t^{[l]}\left([Y_{l-1}]_*^{t-1}\right)$$

- Block decoding @ destination

$$\widehat{M} = g\left([Y_L]_*^{\tau_L + T_{\text{dur}}}\right)$$

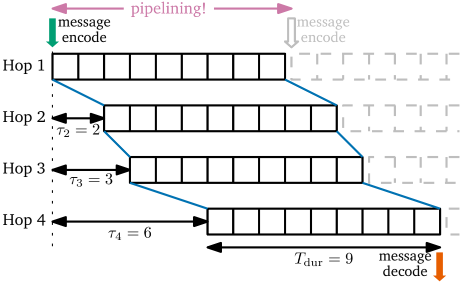
Definition: A (T, R, ϵ)-tuple is achievable by relaying scheme Φ if

(i) $T \geq \tau_L + T_{\text{dur}}$ (ii) $R \leq \dfrac{\log(|\mathcal{M}|)}{T_{\text{dur}}}$ (iii) $\epsilon \geq P(M \neq \widehat{M})$

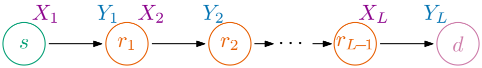
Starting times, relay encoding & joint decoding

A relaying scheme is defined by

- Message $M \in \mathcal{M} \triangleq \{1, \ldots, |\mathcal{M}|\}$
- L deterministic starting times $\tau_1 = 0 \leq \tau_2 \leq \cdots \leq \tau_L$
- Transmission duration T_{dur}



- Coding scheme



- Block encoding @ source

$$X_1(t) = f_t^{[1]}(M)$$

- Causal-sequential coding @ relays

$$Y_l(t) = \mathrm{DMC}_l\big(X_l(t)\big)$$

$$X_l(t) = f_t^{[l]}([Y_{l-1}]_*^{t-1})$$

- Block decoding @ destination

$$\widehat{M} = g([Y_L]_*^{\tau_L + T_{\mathrm{dur}}})$$

Definition: A (T, R, ϵ)-tuple is achievable by relaying scheme Φ if (i) $T \geq \tau_L + T_{\mathrm{dur}}$ (ii) $R \leq \frac{\log(|\mathcal{M}|)}{T_{\mathrm{dur}}}$ (iii) $\epsilon \geq P(M \neq \widehat{M})$

Pipelining:
Source sends next message before previous message is received by destination

Error exponent of relaying scheme Φ

Definition: A (T, R, ϵ)-tuple is achievable by relaying scheme Φ if

(i) $T \geq \tau_L + T_{\text{dur}}$

(ii) $R \leq \dfrac{\log(|\mathcal{M}|)}{T_{\text{dur}}}$

(iii) $\epsilon \geq P(M \neq \widehat{M})$

Error exponent of relaying scheme Φ

Definition: A (T, R, ϵ)-tuple is achievable by relaying scheme Φ if

(i) $T \geq \tau_L + T_{\text{dur}}$

(ii) $R \leq \dfrac{\log(|\mathcal{M}|)}{T_{\text{dur}}}$

(iii) $\epsilon \geq P(M \neq \widehat{M})$

- Let $\mathcal{A}_\phi \triangleq \{\text{all } (T, R, \epsilon) \text{ achievable by } \Phi\}$

Error exponent of relaying scheme Φ

Definition: A (T, R, ϵ)-tuple is achievable by relaying scheme Φ if

(i) $T \geq \tau_L + T_{\text{dur}}$

(ii) $R \leq \dfrac{\log(|\mathcal{M}|)}{T_{\text{dur}}}$

(iii) $\epsilon \geq P(M \neq \widehat{M})$

- Let $\mathcal{A}_\phi \triangleq \{\text{all } (T, R, \epsilon) \text{ achievable by } \Phi\}$
- Then define the error exponent of an L-hop relaying scheme Φ as

$$E_\Phi(R) \triangleq \limsup_{T\to\infty} \sup_{\epsilon:(T,R,\epsilon)\in\mathcal{A}_\Phi} \frac{-\log(\epsilon)}{T}$$

Aside: decode-&-forward, amplify-&-forward

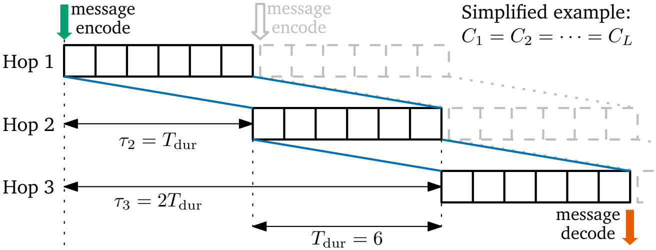
Definitions & analysis framework captures many possible relaying schemes, examples:

Dennis Ogbe, Chih-Chun Wang, David J. Love

Aside: decode-&-forward, amplify-&-forward

Definitions & analysis framework captures many possible relaying schemes, examples:

decode-&-forward

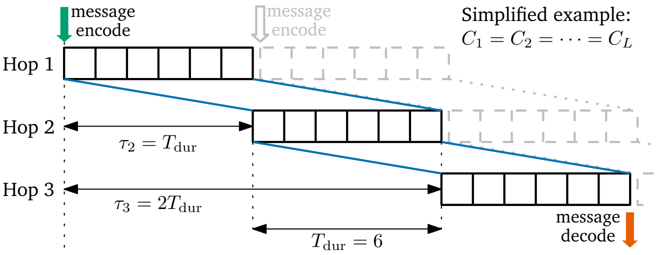


- Random coding over every hop
- T_{dur} is the maximum of the block lengths

Aside: decode-&-forward, amplify-&-forward

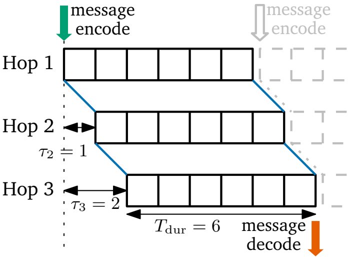
Definitions & analysis framework captures many possible relaying schemes, examples:

decode-&-forward



- Random coding over every hop
- T_{dur} is the maximum of the block lengths

amplify-&-forward

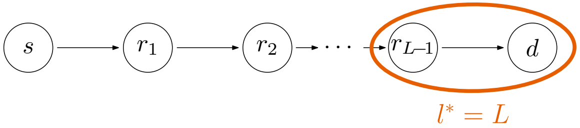


- Random coding between source and destination
- Relays forward received symbols

Dennis Ogbe, Chih-Chun Wang, David J. Love

Main Result 1: Feedback-free setting

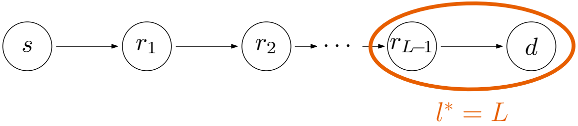
Main Result 1: Feedback-free setting



In the feedback-free setting, **if the bottleneck hop is the last hop** (i.e. $l^* = L$), then transcoding achieves

$$\mathrm{DAF}_{\mathrm{TC}} = 1 \leq \mathrm{DAF}_{\Phi}, \ \forall \Phi.$$

Main Result 1: Feedback-free setting



In the feedback-free setting, **if the bottleneck hop is the last hop** (i.e. $l^* = L$), then transcoding achieves

$$\mathrm{DAF}_{\mathrm{TC}} = 1 \leq \mathrm{DAF}_{\Phi}, \ \forall \Phi.$$

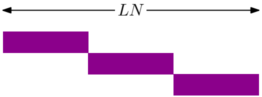
Proof includes:

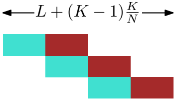
- Description of coding scheme (source, relays, destination)
- Derivation of error probability & delay $\Rightarrow$ Error exponent $\Rightarrow$ DAF

Feedback-free setting — proof outline I

To reduce delay: Break codewords into K micro-blocks, each of length Δ symbols

- Each protected by random coding with rate C_L
- Decode-&-forward over the first $L-1$ hops

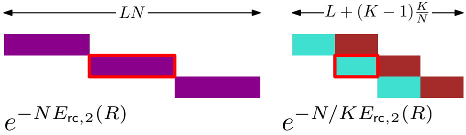




Feedback-free setting — proof outline I

To reduce delay: Break codewords into K micro-blocks, each of length Δ symbols

- Each protected by random coding with rate C_L
- Decode-&-forward over the first $L-1$ hops

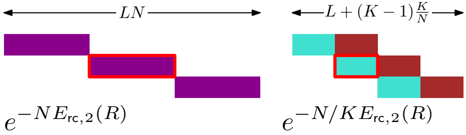


Problem: Error explodes!

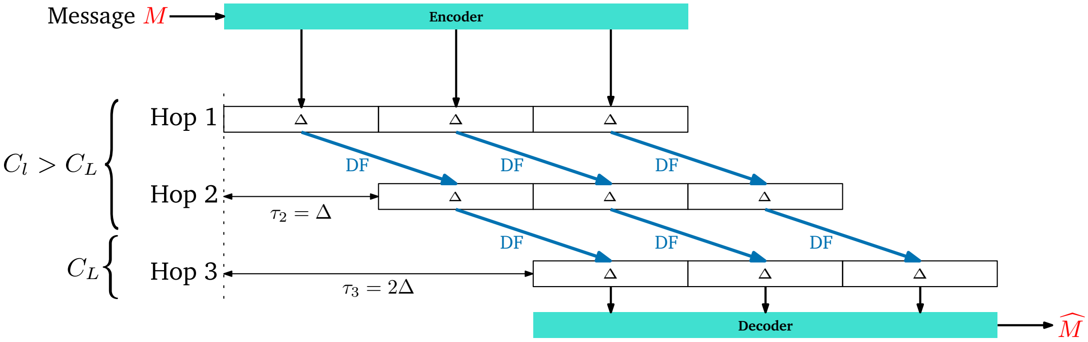
Feedback-free setting — proof outline I

To reduce delay: Break codewords into K micro-blocks, each of length Δ symbols

- Each protected by random coding with rate C_L
- Decode-&-forward over the first $L-1$ hops



Problem: Error explodes!

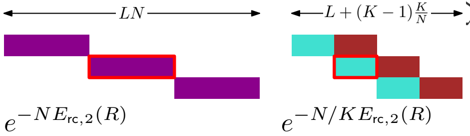


(Example: $L=3$, $K=3$)

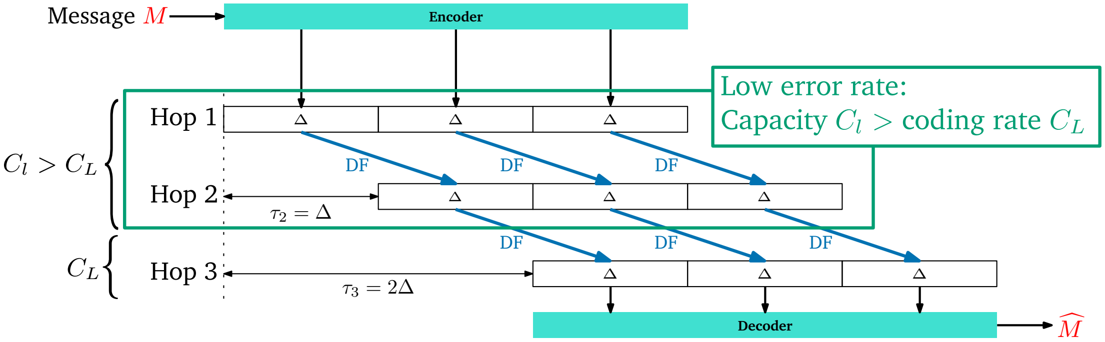
Feedback-free setting — proof outline I

To reduce delay: Break codewords into K micro-blocks, each of length Δ symbols

- Each protected by random coding with rate C_L
- Decode-&-forward over the first $L-1$ hops



Problem: Error explodes!

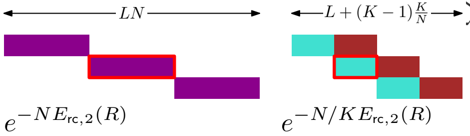


(Example: $L=3$, $K=3$)

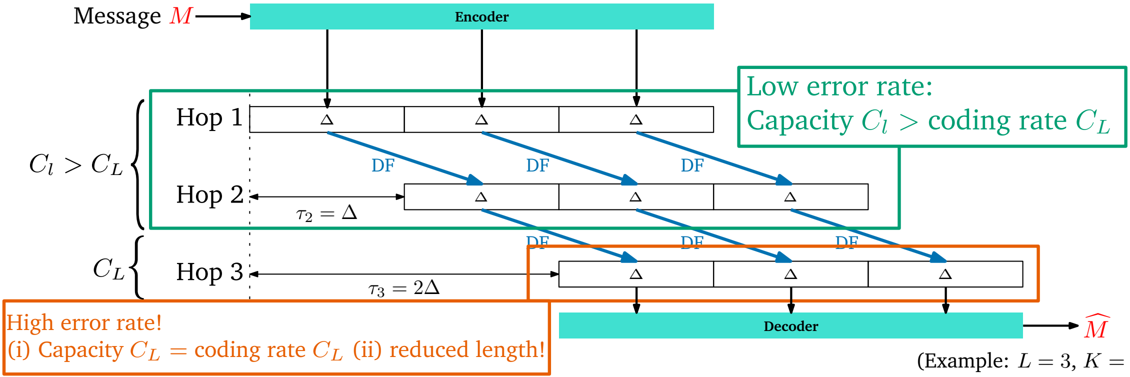
Feedback-free setting — proof outline I

To reduce delay: Break codewords into K micro-blocks, each of length Δ symbols

- Each protected by random coding with rate C_L
- Decode-&-forward over the first $L-1$ hops



Problem: Error explodes!



(Example: $L=3$, $K=3$)

Feedback-free setting — proof outline II

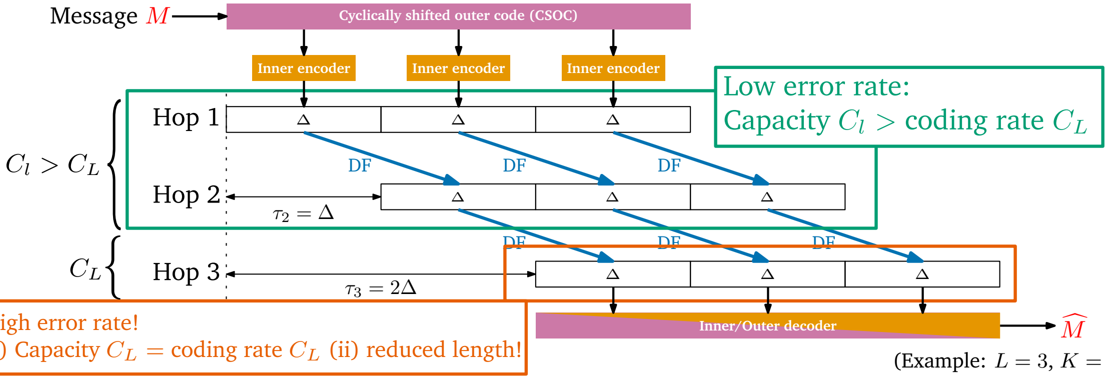
Protect message over last hop using additional coding

- Inspired by concatenated coding
- CSOC outer code + random codes for micro-blocks
- **Maximum-likelihood decoding at destination**

Cyclically shifted outer code (CSOC):

$$\{(\mathbf{c}_{i_1}^{[1]}, \cdots, \mathbf{c}_{i_K}^{[K]}) : \sum_{k=1}^{K} i_k \mod e^{K\Delta(R_I - R)} = 0\}$$

Number of messages: $e^{K\Delta R_I} / e^{K\Delta(R_I - R)} = e^{K\Delta R}$



(Example: $L = 3$, $K = 3$)

Feedback-free setting — proof outline II

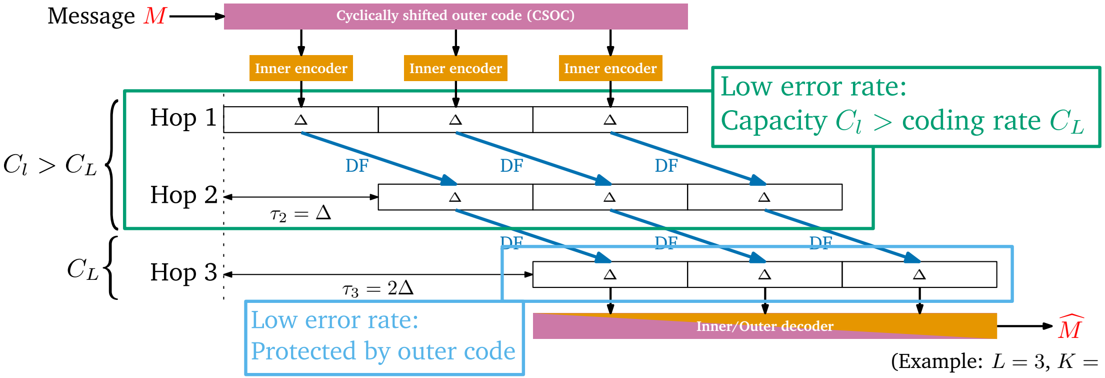
Protect message over last hop using additional coding

- Inspired by concatenated coding
- CSOC outer code + random codes for micro-blocks
- **Maximum-likelihood decoding at destination**

Cyclically shifted outer code (CSOC):

$$\{(\mathbf{c}_{i_1}^{[1]}, \cdots, \mathbf{c}_{i_K}^{[K]}) : \sum_{k=1}^{K} i_k \mod e^{K\Delta(R_I - R)} = 0\}$$

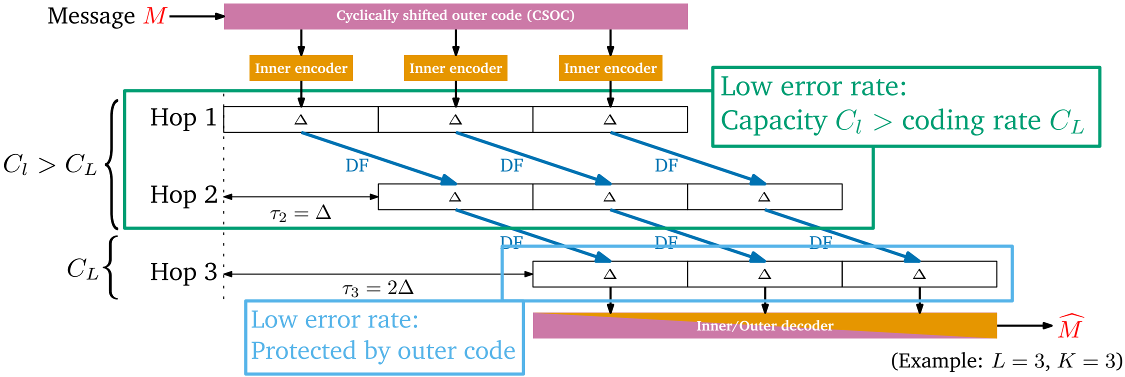
Number of messages: $e^{K\Delta R_I} / e^{K\Delta(R_I - R)} = e^{K\Delta R}$



(Example: $L = 3$, $K = 3$)

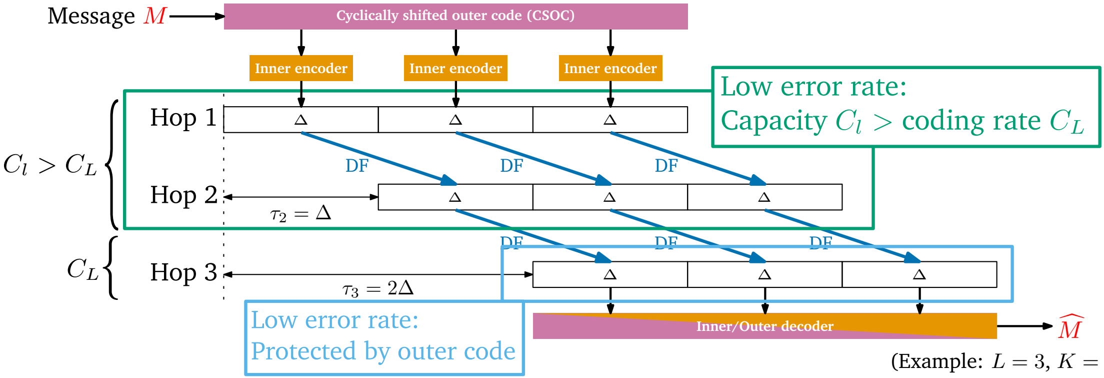
Feedback-free setting — proof outline III

- Choose K such that
  (i) $K \cdot E_{\mathrm{rc},L}(R) < \min_{l \in [1, L-1]} E_{\mathrm{rc},l}(C_L)$
  (ii) $K \cdot (C_L - R) \le C_L$ for any $R < C_L$.



Feedback-free setting — proof outline III

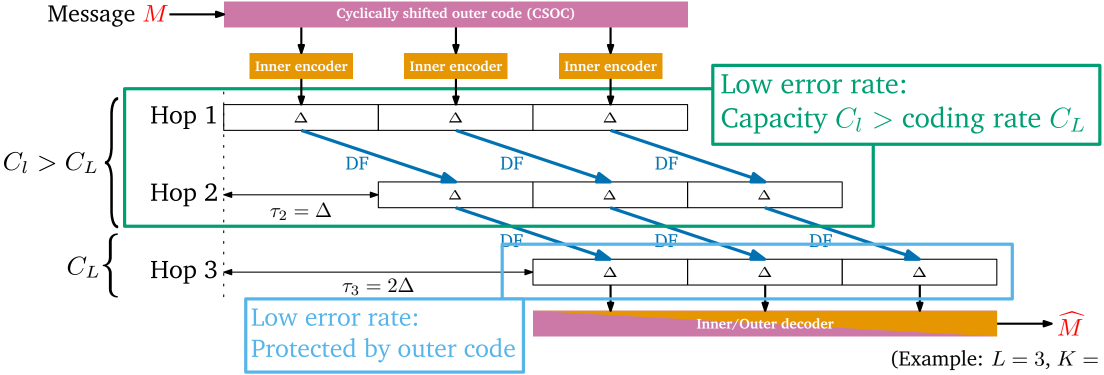
- Choose K such that
  (i) $K \cdot E_{\mathsf{rc},L}(R) < \min_{l \in [1, L-1]} E_{\mathsf{rc},l}(C_L)$
  (ii) $K \cdot (C_L - R) \leq C_L$ for any $R < C_L$.
- Then error probability $\epsilon \leq (K(L-1) + 2^K - 1)e^{-K\Delta E_{\mathsf{rc},L}(R)}$
- Total delay $T = (L - 1 + K)\Delta$



(Example: $L = 3$, $K = 3$)

Feedback-free setting — proof outline III

- Choose K such that
  (i) $K \cdot E_{\text{rc},L}(R) < \min_{l \in [1, L-1]} E_{\text{rc},l}(C_L)$
  (ii) $K \cdot (C_L - R) \leq C_L$ for any $R < C_L$.
- Then error probability $\epsilon \leq (K(L-1) + 2^K - 1)e^{-K\Delta E_{\text{rc},L}(R)}$
- Total delay $T = (L - 1 + K)\Delta$
- Then $E_{\text{TC}}(R) = \frac{K}{K+L-1} E_{\text{rc},L}(R)$



(Example: $L = 3$, $K = 3$)

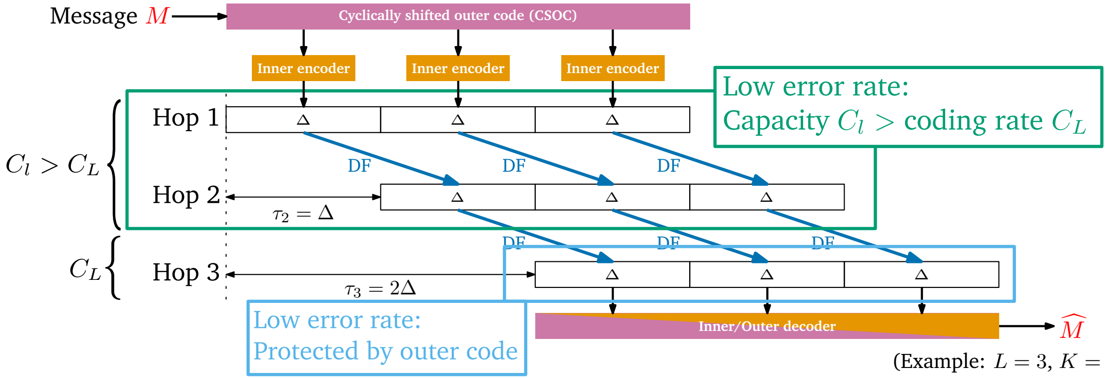
Feedback-free setting — proof outline III

- Choose K such that
  (i) $K \cdot E_{\mathsf{rc},L}(R) < \min_{l\in[1,L-1]} E_{\mathsf{rc},l}(C_L)$
  (ii) $K \cdot (C_L - R) \leq C_L$ for any $R < C_L$.
- Then error probability $\epsilon \leq (K(L-1) + 2^K - 1)e^{-K\Delta E_{\mathsf{rc},L}(R)}$
- Total delay $T = (L - 1 + K)\Delta$

- Then $E_{\mathrm{TC}}(R) = \frac{K}{K+L-1} E_{\mathsf{rc},L}(R)$

Then, when $R \to C_L$, $K \to \infty$ and we get

$$E_{\mathrm{TC}}(R) = E_{\mathsf{rc},L}(R) \quad \blacksquare$$



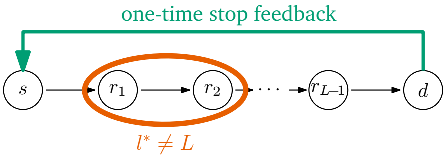
(Example: $L = 3$, $K = 3$)

Main Result 2: Stop feedback setting

Main Result 2: Stop feedback setting

Some new definitions:

- Allow T_{dur} to be a stopping time, use $E[T_{\text{dur}}]$
- Use random coding error exponent with stop feedback [Polyanskiy, Poor, Verdu 2011][2]



$$E_{\mathsf{sf},l}(R) = (C_l - R)^+$$

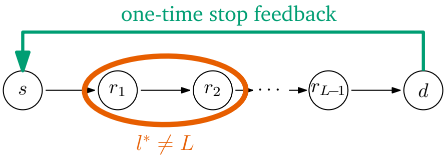
- Then $$\overline{\mathrm{DAF}}_\Phi \triangleq \lim_{R \nearrow C_{l^*}} \frac{E_{\mathsf{sf},l}(R)}{E_\Phi(R)} = \lim_{R \nearrow C_{l^*}} \frac{C_{l^*} - R}{E_\Phi(R)}$$

[2] Y. Polyanskiy, H. V. Poor, and S. Verdu, "Feedback in the non-asymptotic regime," *IEEE Trans. Inform. Theory*, vol. 57, no. 8, pp. 4903–4925, Aug. 2011.

Main Result 2: Stop feedback setting

Some new definitions:

- Allow T_{dur} to be a stopping time, use $E[T_{\text{dur}}]$
- Use random coding error exponent with stop feedback [Polyanskiy, Poor, Verdu 2011]



$$E_{\mathsf{sf},l}(R) = (C_l - R)^+$$

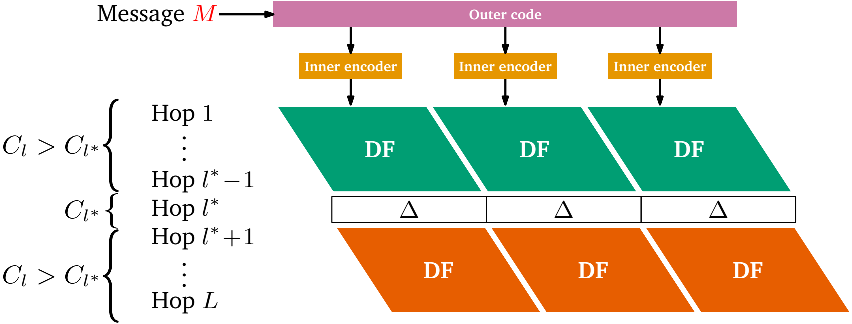
- Then $\overline{\text{DAF}}_\Phi \triangleq \lim_{R \nearrow C_{l^*}} \frac{E_{\mathsf{sf},l}(R)}{E_\Phi(R)} = \lim_{R \nearrow C_{l^*}} \frac{C_{l^*} - R}{E_\Phi(R)}$

In the stop feedback setting, **regardless of the position of the bottleneck link**, transcoding achieves

$$\overline{\text{DAF}}_{\text{TC}} = 1$$

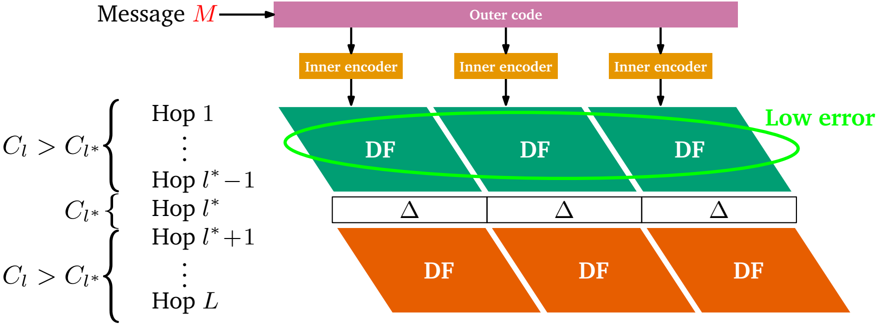
[2] Y. Polyanskiy, H. V. Poor, and S. Verdu, "Feedback in the non-asymptotic regime," *IEEE Trans. Inform. Theory*, vol. 57, no. 8, pp. 4903–4925, Aug. 2011.

Stop feedback setting — proof outline



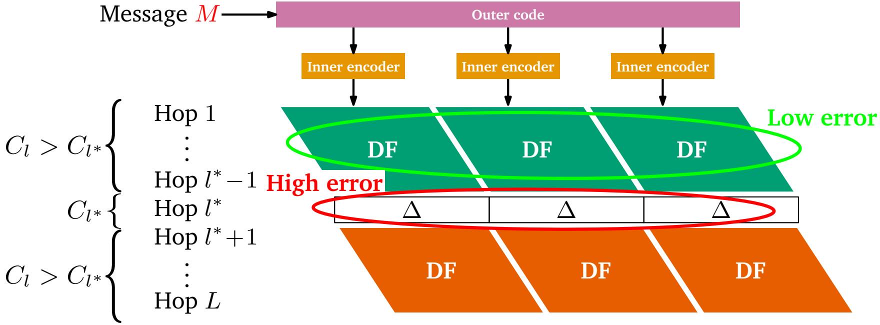
- When $l^* \neq L$, all post-bottleneck hops have high error rate due to error propagation

Stop feedback setting — proof outline



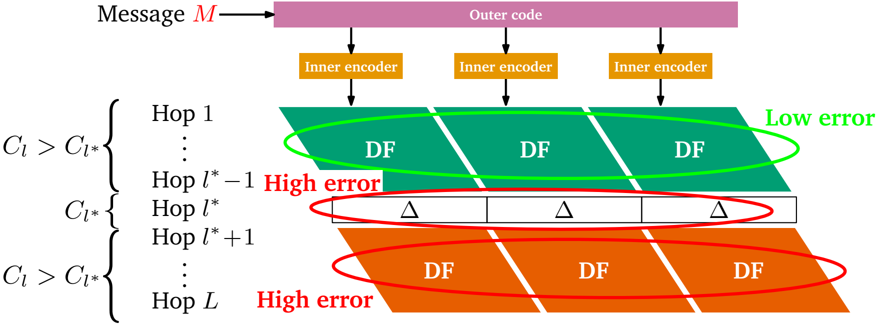
- When $l^* \neq L$, all post-bottleneck hops have high error rate due to error propagation

Stop feedback setting — proof outline



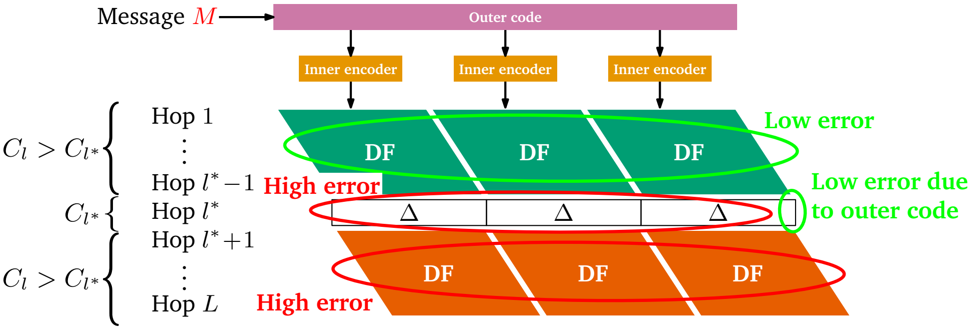
- When $l^* \neq L$, all post-bottleneck hops have high error rate due to error propagation

Stop feedback setting — proof outline



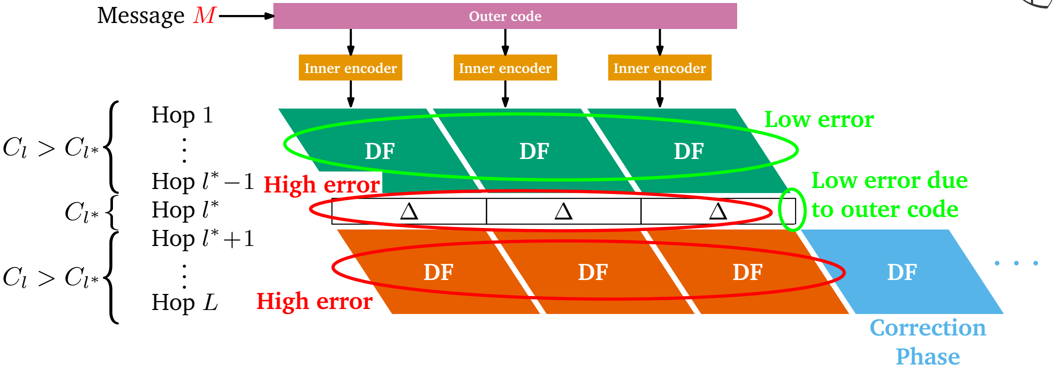
- When $l^* \neq L$, all post-bottleneck hops have high error rate due to error propagation

Stop feedback setting — proof outline



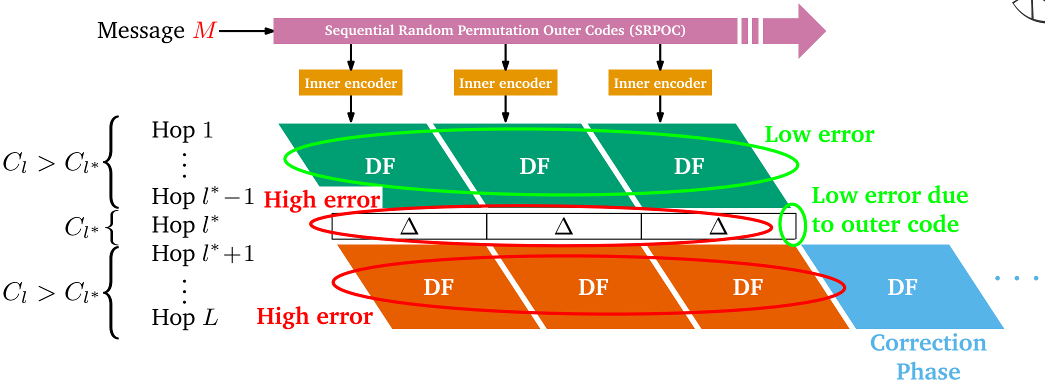
- When $l^* \neq L$, all post-bottleneck hops have high error rate due to error propagation
- Bottleneck relay (BR) can decode outer code after K micro-blocks

Stop feedback setting — proof outline



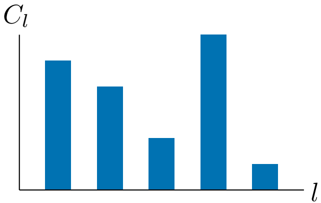
- When $l^* \neq L$, all post-bottleneck hops have high error rate due to error propagation
- Bottleneck relay (BR) can decode outer code after K micro-blocks
- BR sends additional redundancy until destination signals stop feedback

Stop feedback setting — proof outline



- When $l^* \neq L$, all post-bottleneck hops have high error rate due to error propagation
- Bottleneck relay (BR) can decode outer code after K micro-blocks
- BR sends additional redundancy until destination signals stop feedback
- With careful code construction can show $\overline{\text{DAF}}_{\text{TC}} = 1$

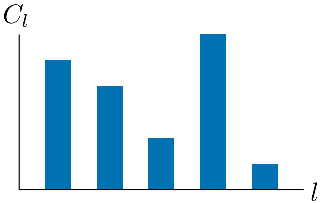
Work in progress



With stop feedback: Solved

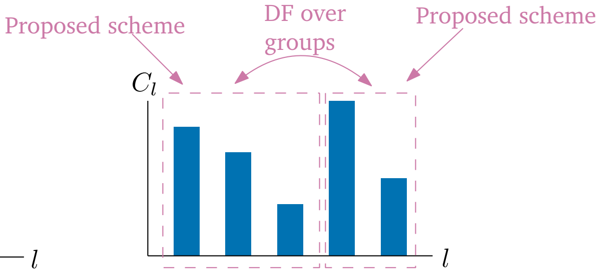
Feedback-free: Solved

Work in progress



With stop feedback: Solved

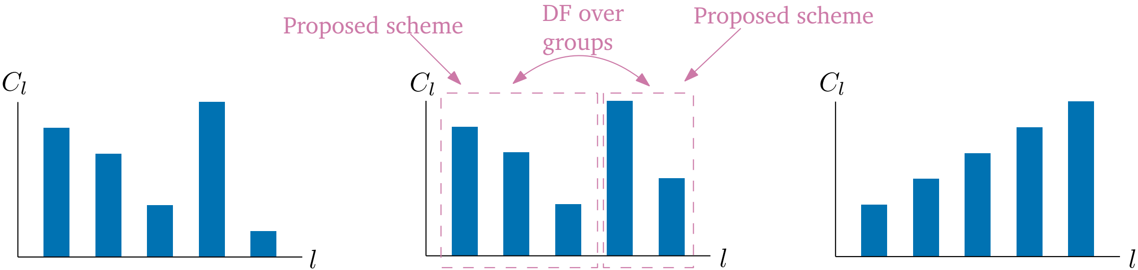
Feedback-free: Solved



With stop feedback: Solved

Feedback-free: Partially solved

Work in progress



With stop feedback: Solved

Feedback-free: Solved

With stop feedback: Solved

Feedback-free: Partially solved

With stop feedback: Solved

Feedback-free: ?

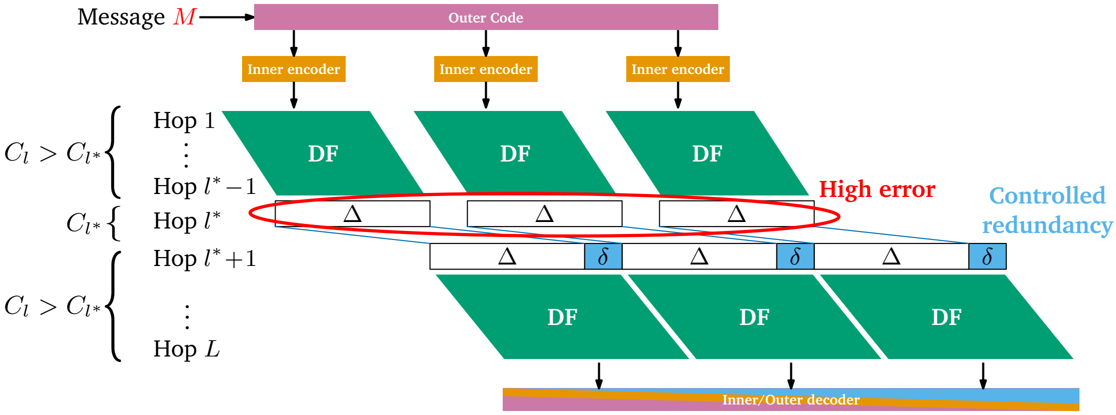
Can we do better than DF?

Work in progress

What is left? — Case $l^* \neq L$ without feedback

Work in progress

What is left? — Case $l^* \neq L$ without feedback

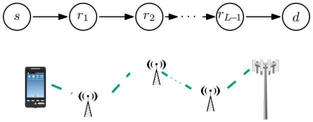


- BR adds redundancy per micro-block
- Conjecture: $1 < \mathrm{DAF}_{\mathrm{TC}'} < \mathrm{DAF}_{\mathrm{DF}}$

Conclusion

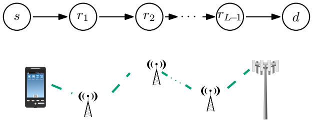
Conclusion

Multi-hop relay channel



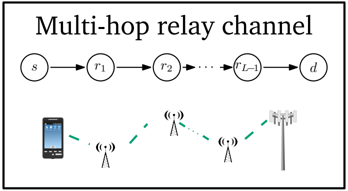
Conclusion

Multi-hop relay channel

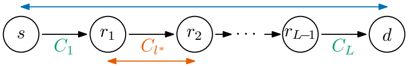


When $R \to C$, what relaying schemes minimize relative delay?

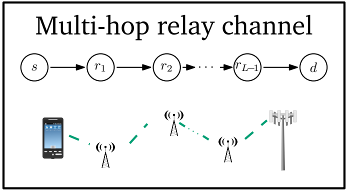
Conclusion



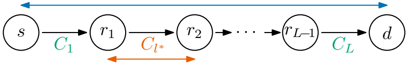
When $R \to C$, what relaying schemes minimize relative delay?

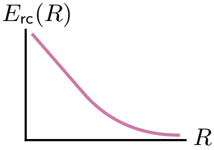


Conclusion

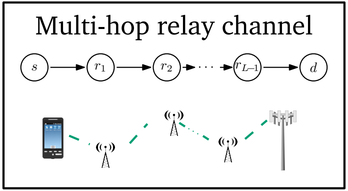


When $R \to C$, what relaying schemes minimize relative delay?

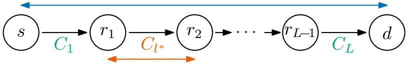


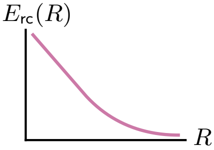


Conclusion



When $R \to C$, what relaying schemes minimize relative delay?

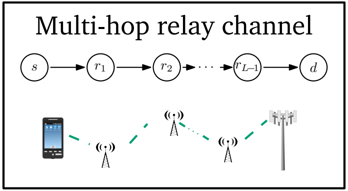




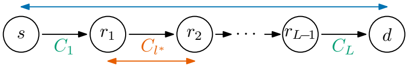
Delay Amplification Factor

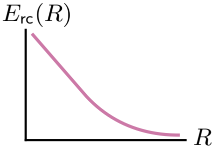
$$\mathrm{DAF}_\Phi \triangleq \lim_{R \nearrow C} \frac{E_{\mathsf{rc},l^*}(R)}{E_\Phi(R)}$$

Conclusion



When $R \to C$, what relaying schemes minimize relative delay?





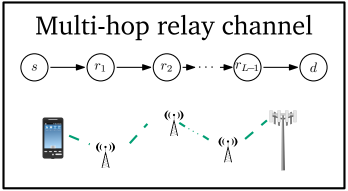
Delay Amplification Factor

$$\mathrm{DAF}_\Phi \triangleq \lim_{R \nearrow C} \frac{E_{\mathsf{rc},l^*}(R)}{E_\Phi(R)}$$

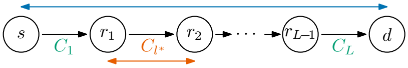
Main Takeaways

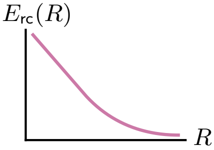
- Relaying schemes with $\mathrm{DAF} = 1$ are possible
- Linearly growing delay of decode-&-forward is NOT a fundamental limit

Conclusion



When $R \to C$, what relaying schemes minimize relative delay?





Delay Amplification Factor

$$\mathrm{DAF}_\Phi \triangleq \lim_{R \nearrow C} \frac{E_{\mathsf{rc},l^*}(R)}{E_\Phi(R)}$$

Main Takeaways

- Relaying schemes with $\mathrm{DAF} = 1$ are possible
- Linearly growing delay of decode-&-forward is NOT a fundamental limit

Future Work

- $l^* \neq L$ without feedback
- Applications to Gaussian, Rayleigh fading channels

Questions.

References I

[1] R. G. Gallager, *Information Theory and Reliable Communication*. New York, NY, USA: John Wiley & Sons, Inc., 1968.

[2] Y. Polyanskiy, H. V. Poor, and S. Verdu, “Feedback in the non-asymptotic regime,” *IEEE Trans. Inform. Theory*, vol. 57, no. 8, pp. 4903–4925, Aug. 2011.

[3] T. Cover and A. E. Gamal, “Capacity theorems for the relay channel,” *IEEE Trans. Inform. Theory*, vol. 25, no. 5, pp. 572–584, Sep. 1979.

[4] R.-A. Pitaval, O. Tirkkonen, R. Wichman, K. Pajukoski, E. Lahetkangas, and E. Tiirola, “Full-duplex self-backhauling for small-cell 5g networks,” *IEEE Wireless Commun.*, vol. 22, no. 5, pp. 83–89, 2015.

[5] H. Ji, S. Park, J. Yeo, Y. Kim, J. Lee, and B. Shim, “Ultra-reliable and low-latency communications in 5g downlink: Physical layer aspects,” *IEEE Wireless Commun.*, vol. 25, no. 3, pp. 124–130, Jun. 2018.

[6] C. Shannon, R. Gallager, and E. Berlekamp, “Lower bounds to error probability for coding on discrete memoryless channels. ii,” *Information and Control*, vol. 10, no. 5, pp. 522–552, 1967.

[7] G. D. Forney, *Concatenated codes.* Cambridge, MA, USA: MIT Press, 1966.

[8] M. V. Burnashev, “Data transmission over a discrete channel with feedback. random transmission time,” *Problems Inform. Transmission*, vol. 12, no. 4, pp. 10–30, 1976.

[9] S. L. Fong and V. Y. F. Tan, “Achievable rates for gaussian degraded relay channels with non-vanishing error probabilities,” *IEEE Trans. Inform. Theory*, vol. 63, no. 7, pp. 4183–4201, Jul. 2017.

[10] Y. Hu, J. Gross, and A. Schmeink, “On the capacity of relaying with finite blocklength,” *IEEE Trans. Veh. Technol.*, vol. 65, no. 3, pp. 1790–1794, Mar. 2016.

References II

[11] V. Y. F. Tan, “On the reliability function of the discrete memoryless relay channel,” *IEEE Trans. Inform. Theory*, vol. 61, no. 4, pp. 1550–1573, Apr. 2015.

[12] N. Wen and R. A. Berry, “Reliability constrained packet-sizing for linear multi-hop wireless networks,” in *2008 IEEE International Symposium on Information Theory*, Jul. 2008, pp. 16–20.

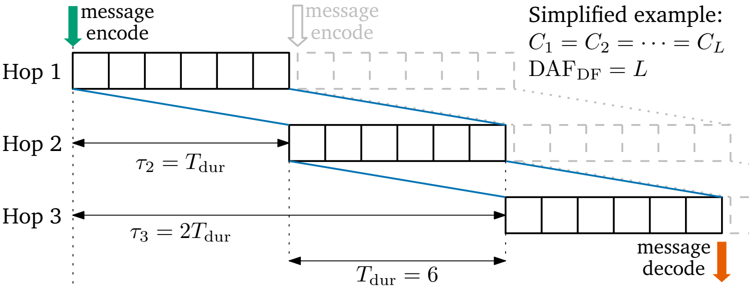
[13] C. Thommesen, “Error-correcting capabilities of concatenated codes with MDS outer codes on memoryless channels with maximum-likelihood decoding,” *IEEE Trans. Inform. Theory*, vol. 33, no. 5, pp. 632–640, Sep. 1987.

[14] C.-C. Wang, D. J. Love, and D. Ogbe, “Transcoding: A new strategy for relay channels,” in *55th Annual Allerton Conference on Communication, Control, and Computing (Allerton)*, Oct. 2017.

[15] Y. Polyanskiy, H. V. Poor, and S. Verdu, “Channel coding rate in the finite blocklength regime,” *IEEE Trans. Inform. Theory*, vol. 56, no. 5, pp. 2307–2359, May 2010.

Backup Slides.

Aside: DAF of decode-&-forward



Simplified example:
$C_1 = C_2 = \cdots = C_L$
$\mathrm{DAF}_{\mathrm{DF}} = L$

- Operate each hop close to capacity $\Rightarrow T \approx \sum_l \frac{B}{C_l}$ for a B-bit message
- But $T_{bn} \approx \frac{B}{C_{l^*}}$
- Then $\mathrm{DAF}_{\mathrm{DF}} = \frac{T}{T_{bn}} = \sum_l \frac{C_{l^*}}{C_l} > 1$

N.B.: AF does not achieve capacity, DAF does not apply

Prior Work: Transcoding